A COMPARISON FOR THE RELIABILITY OF THE HOLLOW AND SOLID CIRCULAR SHAFT SUBJECTED TO TORSIONAL LOADING ANALYSING SHEAR STRESS BEHAVIOUR UNDER EXPONENTIAL AND NORMAL DISTRIBUTION

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ABSTRACT

Torsion is the process of a straight component twisting under the influence of a torque, which aims to induce rotation about its longitudinal axis. The generated stress is random in nature statistical approach has been considered. This study presents cooperate reliability of shafts subjected to torsion, focusing on both hollow and solid circular shafts, assuming that shear stress follows exponential and normal distributions and reliability has been computed and compared by changing the parameters.

Keywords: Reliability, shear stress, normal distribution, exponential distribution, circular solid shaft, hollow circular shaft.

1. INTRODUCTION

Shaft is a critical component of the power transmission system, requiring meticulous design to ensure the effective operation of machinery. It is the rotating element that transfers power from one point to another. As a fundamental part of nearly all mechanical machines and systems, the shaft plays a pivotal role in their functionality. Anil Misra et al. [1], discussed with the view of developing methods for reliability based design, the finite difference technique was combined with the Monte-Carlo simulation method to create a probabilistic load–displacement analysis. The Monte Carlo simulation method was used, in lieu of other closed-form probabilistic techniques, due to the complexity of the load–displacement analysis. Dr. Edward E.Osakue et al. [2] studied a design verification study of three shafts loaded in bending and torsion was conducted based on probabilistic approach using Gerber failure rule. The probabilistic model parameters like mean and variance values to estimate expected design results while then reliability of the design is evaluated using the coefficients of variation or covariance of the design parameters. The equations for the covariance of strength and stress parameters were developed using sensitivity analysis based on first order Taylor series expansion of design relationships. Dr. R. K. Bansal [3] discussed shear stress produced in a circular shaft subjected to torsion, Torque transmitted by a circular solid shaft and a hollow circular shaft. E. Balagurusamy [4] discussed stress dependent hazard models, the failure rate of almost all components is stress dependent. K. C. Kapur and L. R. Lamberson [5] discussed design of a shaft subjected to torsion, when a shaft is subjected to a torque a shearing stress is produced in the shaft. The shear stress varies from zero in the axis to a maximum at the outside surface of the shaft. P. Hari Prasad et al. [6] studied eccentricity of symmetrical column in a widely varying range of values. The increase in eccentricity was not reflective of the increase in reliability. T. S. Uma Maheswari et al. [7] conducted a reliability analysis of unsymmetrical columns subjected to eccentric loads, assuming that stress follows an exponential distribution. Pasha, M. Y., et al. [9] performed a reliability analysis of a shaft subjected to a twisting moment and a bending moment, considering normally distributed strength and stress. Yakoob Pasha, M., et al. [10] studied the reliability analysis of a shaft subjected to a twisting moment and a bending moment for exponentially and Weibull-distributed strength and stress.

2. Torsion of shafts

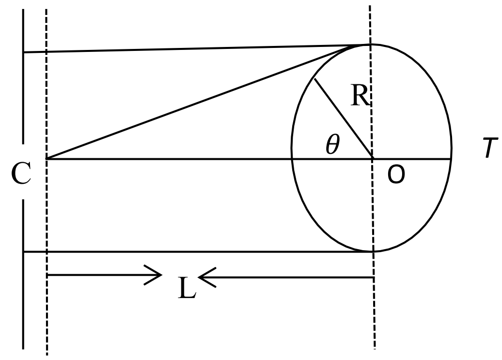
The shaft is said to be in torsion when equal and opposing torques are applied to the shaft's two ends. The torque is calculated by multiplying the force (tangential to the shaft ends) by the shaft's radius.

The shaft experiences a twisting moment as a result of the torques applied at the two ends. The material of the shaft is subjected to shear loads and strains as a result.

2.1 Shear stress produced in a circular shaft subjected to torsion

If R is radius of the shaft, C is modulus of rigidity, θ is angle of twisting moment and L is length of the shaft then the maximum shear stress (τ) induced in a shaft subjected to twisting moment [3] is given by

$$\frac{\tau}{R} = \frac{C \times \theta}{L} \Rightarrow \tau = \frac{R \times C \times \theta}{L} \quad (1)$$

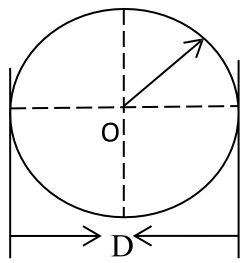


2.2 Torque transmitted by a circular solid shaft

If τ is maximum shear stress and D is diameter of circular solid shaft then the maximum torque transmitted by a circular solid shaft is obtained from the maximum shear stress induced at the outer surface of the solid shaft.

The torque(T)transmitted by a solid shaft [3] is given by

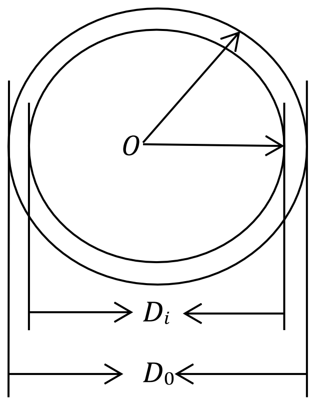
$$T = \frac{\square}{16} \times \tau \times D^3 \Rightarrow \tau = \frac{16 \times T}{\pi \times D^3} \quad (2)$$



2.3 Torque transmitted by a hollow circular shaft

If τ is maximum shear stress, D_0 is external diameter of hollow circular shaft and D_i is internal diameter of hollow circular shaft then the Torque transmitted by a hollow circular shaft is obtained in the same way as for a solid shaft. The torque (T) transmitted by hollow circular shaft [9] is given by

$$T = \frac{\pi \times \tau}{16}\left[\frac{D_0^4 - D_i^4}{D_0}\right]$$



$$\tau = \frac{16 \times T \times D_0}{\pi \times (D_0^4 - D_i^4)} \qquad (3)$$

3. Statistical Methodology

The probability of failure as function of time t can be defined by

$$(t) = P(t' \leq t), \qquad t \geq 0.$$

where t' is a random variable denoting the time to failure. The reliability function is

$$(t) = 1 - F(t) = P(t' \geq t), \qquad t \geq 0.$$

The failure rate of the almost all components is stress dependent. For such cases, a power function model is defined [9] as given below

$$z(t) = h(t) \times \sigma_1^a \times \sigma_2^b$$

where $z(t)$ is the failure rate at rated stress conditions, $h(t)$ is hazard function, σ_1 and σ_2 are stress ratios for two different kind of stresses and a, b are positive constants.

The normal distribution takes the well-known bell shape. This distribution is symmetrical about its mean value. The probability density function for a normally distributed stress χ and normally distributed strength ξ is given by [5]

$$f_z(\chi) = \frac{1}{\sigma_z\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\chi - \mu_z}{\sigma_z}\right)^2\right] \text{ for } -\infty < \chi < \infty$$

$$f_\xi(\xi) = \frac{1}{\sigma_\xi\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{\xi - \mu_\xi}{\sigma_\xi}\right)^2\right] \text{ for } -\infty < \xi < \infty$$

where μ_z is mean value of the stress, σ_z is standard deviation of the stress, μ_ξ is mean value for the strength, and σ_ξ is standard deviation of the strength.
If $y = \xi - \chi$, then it is well known that the random variable y is normally distributed with mean of $\mu_y = \mu_\xi - \mu_z$ and standard deviation of $\sigma_y = \sqrt{\sigma_\xi^2 + \sigma_z^2}$.

The reliability R can be expressed in terms of y as

$$R = (y > 0) = \frac{1}{\sigma_y\sqrt{2\pi}} \int_0^\infty \exp\left[-\frac{1}{2}\left(\frac{y - \mu_y}{\sigma_y}\right)^2\right] dy$$

If $z = (y - \mu_y)/\sigma_y$, then $\sigma_y\, dz = dy$.
when $y = 0$, the lower limit of z is given by

$$z = \frac{0 - \mu_y}{\sigma_y} = -\frac{(\mu_\xi - \mu_z)}{\sqrt{\sigma_\xi^2 + \sigma_z^2}}$$

and $y \to +\infty$, the upper limit of $z \to +\infty$.
Then the reliability is given by

$$R = \frac{1}{\sqrt{2\pi}} \int_{-\frac{(\mu_\xi - \mu_z)}{\sqrt{\sigma_\xi^2 + \sigma_z^2}}}^{\infty} \exp\left(\frac{-z^2}{2}\right) dz \qquad (4)$$

The random variable $z = \frac{(y - \mu_y)}{\sigma_y}$ is the standard normal variable.
The exponential distribution is the probability distribution of the time between events in a process in which events occur continuously and independently at a constant average rate. Its probability density function is given by

$$f(t) = \frac{1}{\lambda}\exp\left(-\frac{t}{\lambda}\right)$$

where $t \geq 0$ and $\lambda > 0$ is the scale parameter.
The corresponding cumulative distribution function is given by

$$(t) = 1 - \exp\left(-\frac{t}{\lambda}\right) \text{ for } t \geq 0$$

and the reliability function

$$(t) = \exp\left(-\frac{t}{\lambda}\right) \text{ for } t \geq 0$$

The formula for the hazard function of the exponential distribution is $h(t) = \frac{1}{\lambda}$

3.1 The reliability of Shear stress produced in a circular shaft subjected to torsion

The reliability function Shear stress produced in a circular shaft subjected to torsion using normal distribution is

$$R_N = \frac{1}{\sqrt{2\pi}} \int_{\frac{-\left(\mu_\xi - \frac{R\times C\times\theta}{L}\right)}{\sqrt{\sigma_\xi^2+\sigma_z^2}}}^{\infty} \exp\left(\frac{-z^2}{2}\right) dz \quad (5)$$

The reliability function shear stress produced in a circular shaft subjected to torsion using exponential distribution is

$$R_2 = \exp\left[-\int_0^t (t)dt\right]$$

where $z(t) = h(t) \times \tau$, the failure rate function.

Therefore

$$z(t) = \frac{R \times C \times \theta}{\lambda \times L}$$

$$R_E = \exp\left[\frac{-t \times R \times C \times \theta}{\lambda \times L}\right] \quad (6)$$

3.2 The reliability of torque transmitted by a circular solid shaft

The reliability function for the Torque transmitted by a circular solid shaft using normal distribution is

$$R_N = \frac{1}{\sqrt{2\pi}} \int_{\frac{-\left(\mu_\xi - \frac{16\times T}{\pi\times D^3}\right)}{\sqrt{\sigma_\xi^2+\sigma_z^2}}}^{\infty} \exp\left(\frac{-z^2}{2}\right) dz \quad (7)$$

The reliability function for the Torque transmitted by a circular solid shaft using exponential distribution is

$$R_2 = \exp\left[-\int_0^t (t)dt\right]$$

where $z(t) = h(t) \times \tau$, the failure rate function

$$(t) = \frac{16T}{\lambda \times \pi \times D^3}$$

Therefore

$$R_E = \exp\left[\frac{-t \times 16T}{\lambda \times \pi \times D^3}\right] \quad (8)$$

3.2 The reliability of torque transmitted by a hollow circular shaft

The reliability function for torque transmitted by a hollow circular shaft using normal distribution is

$$R_N = \frac{1}{\sqrt{2\pi}} \int_{-\left(\mu_\xi - \frac{16 \times T \times D_0}{\pi \times (D_0^4 - D_i^4)}\right) / \sqrt{\sigma_\xi^2 + \sigma_z^2}}^{\infty} \exp\left(\frac{-z^2}{2}\right) dz \quad (9)$$

The reliability function for Torque transmitted by a hollow circular shaft using exponential distribution is

$$R_2 = \exp\left[-\int_0^t (t)dt\right]$$

where $z(t) = h(t) \times \tau$, the failure rate function.

$$(t) = \frac{16T \times D_0}{\lambda \times \pi \times [D_0^4 - D_i^4]}$$

Therefore

$$R_E = \exp\left[\frac{-t \times 16T \times D_0}{\lambda \times \pi \times (D_0^4 - D_i^4)}\right] \quad (10)$$

4. Reliability computations and discussion

Reliability computations were performed for the shear stress produced in a circular shaft subjected to torsion and for the torque transmitted by both circular solid shafts and hollow circular shafts.

4.1 Shear stress produced in a circular shaft subjected to torsion

Table – 1: R=16mm, C=73.1 Gpa, L=64 mm.

Θ	R_N	R_E
0.06	0.849188	0.193061
0.08	0.809451	0.111582
0.1	0.763866	0.06449
0.12	0.712844	0.037272
0.14	0.657125	0.021542
0.16	0.597758	0.01245
0.18	0.536042	0.007196
0.2	0.473444	0.004159
0.22	0.411497	0.002404
0.24	0.351684	0.001389

Graph 1: Reliability vs Twisting moment

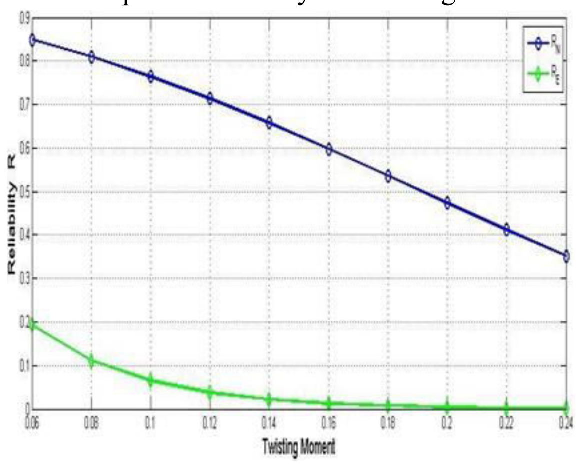


Table – 2: R=16mm, Θ=0.0698 radians, L=32 mm.

C	R_N	R_E
4.2	0.998845	0.940035
16.2	0.996749	0.787793
28.2	0.9917	0.660208
40.2	0.980752	0.553285

52.2	0.959388	0.463679
64.2	0.921864	0.388584
76.2	0.862541	0.325652
88.2	0.77813	0.272912
100.2	0.670025	0.228713
112.2	0.818257	0.191672

Graph 2: Reliability vs Modulus of Tigidity

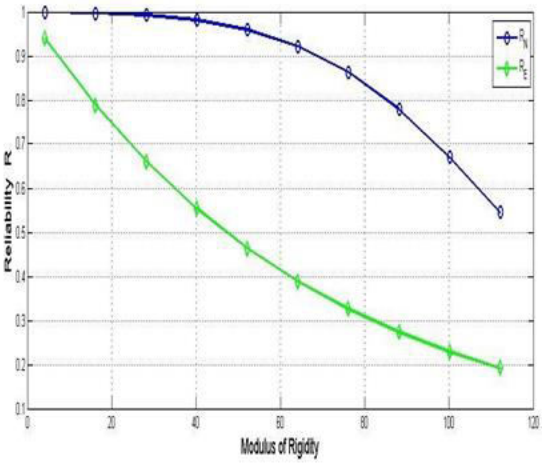
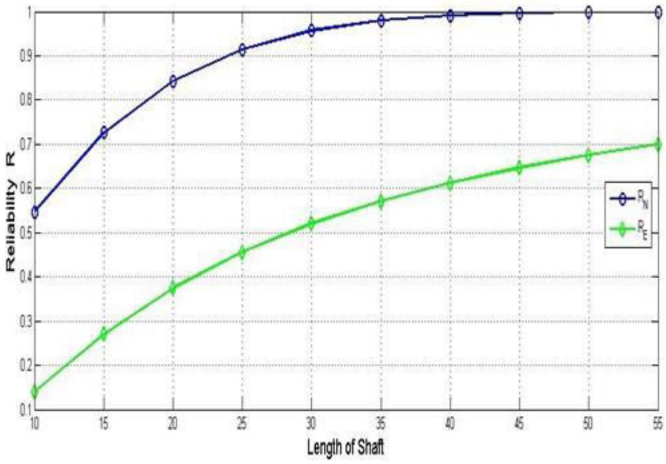


Table – 3: R=16mm, ϴ=0.0698 radians, C=73.1 Gpa.

L	RN	RE
10	0.546091	0.140955
15	0.726643	0.270845
20	0.842261	0.37544
25	0.914746	0.456701
30	0.957301	0.520428
35	0.980301	0.571321
40	0.99166	0.612731
45	0.996768	0.647004
50	0.998856	0.675797
55	0.999631	0.700305

Graph 3: Reliability vs Length of shaft



4.2 Torque transmitted by a circular solid shaft

Table – 4: □□3.1415926, D=198 mm.

T	RN	RE
1000000	0.999522	0.720324549
2000000	0.998524	0.518867456
3000000	0.995883	0.373752966
4000000	0.989612	0.269223437
5000000	0.976245	0.193928251
6000000	0.950676	0.13969128
7000000	0.906775	0.100623058
8000000	0.839125	0.072481259
9000000	0.745558	0.05221003
1E+07	0.629409	0.037608166

Graph 4: Reliability vs Total Torque

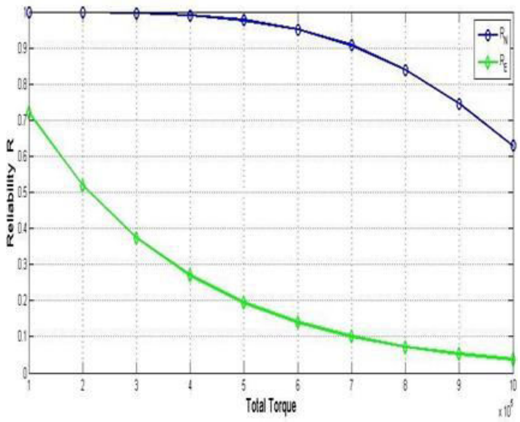
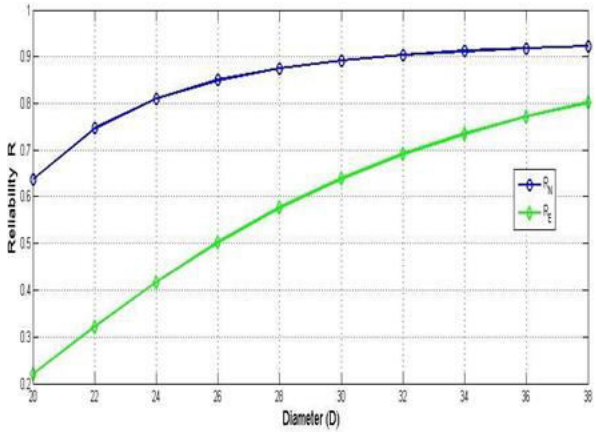


Table – 5: π=3.1415926, T=18900000

D	R_N	R_E
20	0.637791	0.220474776
22	0.746684	0.321111479
24	0.810657	0.416868778
26	0.849684	0.5024805
28	0.874669	0.576367271
30	0.891416	0.638909987
32	0.903102	0.691332914
34	0.911538	0.735099633
36	0.917807	0.771626653
38	0.922578	0.802168805

Graph 5: Reliability vs Diameter



4.2 Torque transmitted by a hollow circular shaft

Table – 6: T=982000, □□□□□□□□□□□□ D_i=142

D_0	R_N	R_E
43	0.518061	0.145039757
45	0.673161	0.201515752
47	0.780997	0.258513915

49	0.853151	0.313980398
51	0.900958	0.366717508
53	0.932712	0.416103505
55	0.95396	0.461886918
57	0.968303	0.504045874
59	0.978069	0.542695144
61	0.984769	0.57802595

Graph 6: Reliability vs Internal Diameter

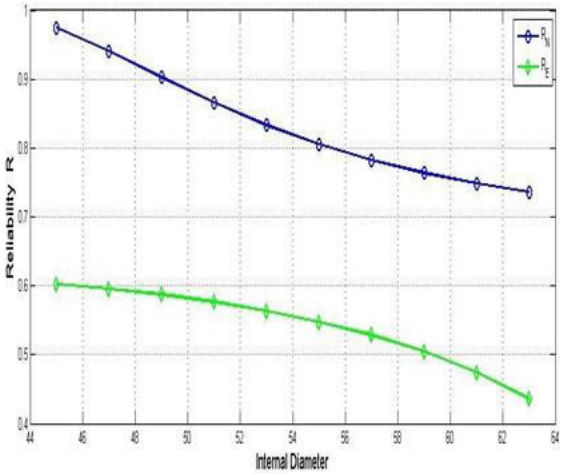
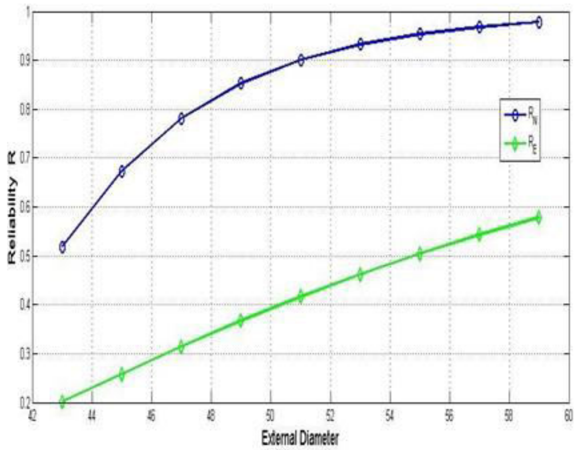


Table – 6: T=982000,□□□□□□□□□□□□ Do=54

Di	RN	RE
45	0.974295	0.602793336
47	0.940747	0.595701501
49	0.902529	0.587040577
51	0.865772	0.576474542
53	0.833205	0.56357275
55	0.805529	0.547772586
57	0.782526	0.528323832
59	0.763648	0.504204203
61	0.748281	0.473989031

63	0.735852	0.435649428

Graph 7: Reliability vs External Diameter



5. CONCLUSION

The torsion reliability of a circular shaft under normally distributed shear stress has been calculated. The reliability of hollow and solid circular shafts in transmitting torque has been assessed. It was found that shaft reliability increases with an increase in the shaft's length, total torque, and diameter. Conversely, reliability decreases with an increase in the twisting moment, modulus of rigidity, and internal diameter.

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